Incompressible Navier-Stokes methods for wind-energy applications

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NWO Applied and Engineering Sciences

EUROS WP1.4: Wind Farm Wake Interactions

Goals:

- More accurate modelling of wakes
- Decrease uncertainty in load predictions

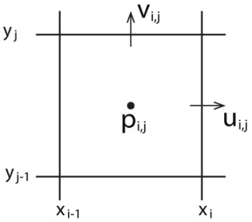
$$
\begin{aligned}
\frac{\partial \boldsymbol{u}}{\partial t} &= -(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + \nu \Delta \boldsymbol{u} - \nabla p && \text{in } \Omega \\
0 &= \nabla \cdot \boldsymbol{u} && \text{in } \Omega
\end{aligned}
$$

Approach:

- Using one Cartesian mesh
- More accurate fluid-rotor blades interaction

MAC scheme

Cartesian staggered grid [Harlow, Welch 1965]



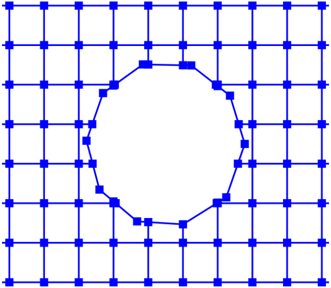
The method of choice

- No spurious pressure modes
- Conserves mass, momentum, kinetic energy, vorticity
- Small stencil: cheap
- Perfect for turbulence modeling (DNS, LES)

Desired: Extension of MAC scheme to non-Cartesian meshes

Extension MAC scheme to polyhedral cells

- Direct modelling of turbine tower and blades in Cartesian mesh
- Excellent for mesh refinement (Cartesian and non-Cartesian)



Mimetic discretization

Two elements for a conservative scheme on polydral meshes

1. Exact discretization of grad, curl, div
   - grad, curl, div are building blocks of NS-equations
   - Correct discretization of grad, curl, div essential for conservation of mass, momentum, vorticity, energy

Mimetic discretization

Two elements for a conservative scheme on polydral meshes

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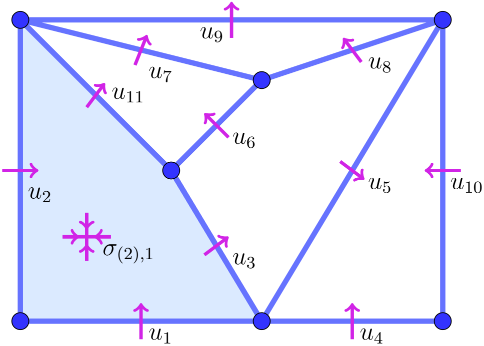
   - grad, curl, div are building blocks of NS-equations
   - Correct discretization of grad, curl, div essential for conservation of mass, momentum, vorticity, energy

2. Dual mesh
   - To derive a square solvable linear system
   - Needed to preserve all symmetries of grad, curl, div

1. EXACT DISCRETIZATION OF GRAD, CURL, DIV

SIMPLE EXAMPLE IN 2D: EXACT DISCRETIZATION OF $\nabla \cdot \underline{u} = 0$

$$u_k := \int_{\sigma_{(1),k}} \underline{u} \cdot \underline{n} \, \mathrm{d}L$$



$$\mathbf{0} = \mathbb{D}^{(2,1)} \boldsymbol{u}^{(1)} = \begin{pmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ & & & & & & & & & & \\ & & & & & & & & & & \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_{11} \end{pmatrix}$$

1. Exact discretization of grad, curl, div

Simple example in 2D: exact discretization of $\nabla \cdot \underline{u} = 0$

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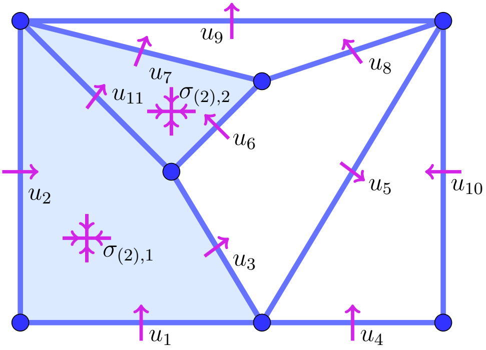

$$\mathbf{0} = \mathbb{D}^{(2,1)} \boldsymbol{u}^{(1)} = \begin{pmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 1 \\ & & & & & & & & & & \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_{11} \end{pmatrix}$$

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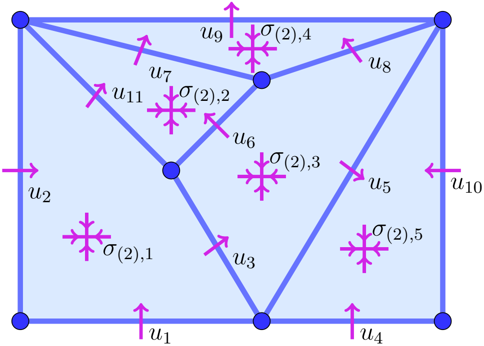

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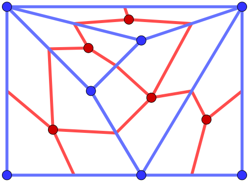
Primal-dual mesh

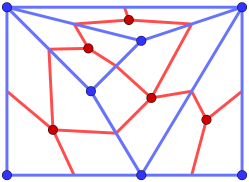
Primal mesh:	$\leftrightarrow$	Dual mesh:
kD cell		$(n-k)$D cell



Primal-dual mesh

Primal mesh: ↔ Dual mesh:

kD cell ↔ $(n-k)$D cell



Primal mesh

$\boldsymbol{u}^{(2)}$

$\downarrow$

$\mathbb{H}^{(2)}\boldsymbol{u}^{(2)}$

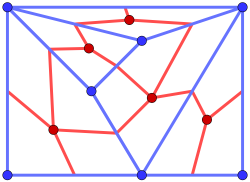
Dual mesh

Interpolate from primal 2D-cells to dual 1D-cells

Primal-dual mesh

Primal mesh: $\leftrightarrow$ Dual mesh:

kD cell $(n-k)$D cell



Primal mesh

$\boldsymbol{u}^{(2)}$

$\downarrow$

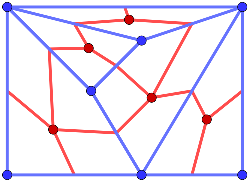
$\tilde{\mathbb{D}}^{(2,1)}\mathbb{H}^{(2)}\boldsymbol{u}^{(2)} \leftarrow \mathbb{H}^{(2)}\boldsymbol{u}^{(2)}$

Dual mesh

Apply curl-incidence matrix on dual mesh

Primal-dual mesh

Primal mesh: kD cell $\leftrightarrow$ Dual mesh: $(n-k)$D cell



$(\mathbb{H}^{(1)})^{-1}\tilde{\mathbb{D}}^{(2,1)}\mathbb{H}^{(2)}\boldsymbol{u}^{(2)}$ — Primal mesh — $\boldsymbol{u}^{(2)}$

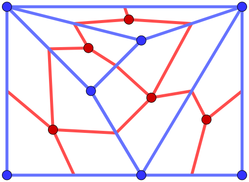
$\uparrow$ $\downarrow$

$\tilde{\mathbb{D}}^{(2,1)}\mathbb{H}^{(2)}\boldsymbol{u}^{(2)} \leftarrow \mathbb{H}^{(2)}\boldsymbol{u}^{(2)}$ — Dual mesh

Interpolate from dual 2D-cells to primal 1D-cells

Primal-dual mesh

Primal mesh: kD cell $\leftrightarrow$ Dual mesh: $(n-k)$D cell



$$(\mathbb{H}^{(1)})^{-1}\tilde{\mathbb{D}}^{(2,1)}\mathbb{H}^{(2)}\boldsymbol{u}^{(2)} \longrightarrow \mathbb{D}^{(2,1)}(\mathbb{H}^{(1)})^{-1}\tilde{\mathbb{D}}^{(2,1)}\mathbb{H}^{(2)}\boldsymbol{u}^{(2)}$$

Primal mesh

$$\boldsymbol{u}^{(2)}$$

$$\uparrow \qquad\qquad \downarrow$$

$$\tilde{\mathbb{D}}^{(2,1)}\mathbb{H}^{(2)}\boldsymbol{u}^{(2)} \longleftarrow \mathbb{H}^{(2)}\boldsymbol{u}^{(2)}$$

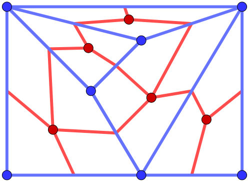
Dual mesh

Apply curl-incidence matrix on primal mesh

Primal-dual mesh

Primal mesh: $\leftrightarrow$ Dual mesh:

kD cell $(n-k)$D cell



$(\mathbb{H}^{(1)})^{-1}\tilde{\mathbb{D}}^{(2,1)}\mathbb{H}^{(2)}\boldsymbol{u}^{(2)} \longrightarrow \mathbb{D}^{(2,1)}(\mathbb{H}^{(1)})^{-1}\tilde{\mathbb{D}}^{(2,1)}\mathbb{H}^{(2)}\boldsymbol{u}^{(2)}$ Primal mesh

$\boldsymbol{u}^{(2)}$

$\uparrow$ $\downarrow$

$\tilde{\mathbb{D}}^{(2,1)}\mathbb{H}^{(2)}\boldsymbol{u}^{(2)} \longleftarrow \mathbb{H}^{(2)}\boldsymbol{u}^{(2)}$ Dual mesh

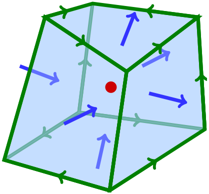
Conservative scheme

Semi-discrete equations: Conserving mass, momentum, energy and vorticity

$\boldsymbol{u}^{(2)}$: normal flux across faces of primal mesh

$\boldsymbol{\omega}^{(1)}$: line integral vorticity along edges of primal mesh

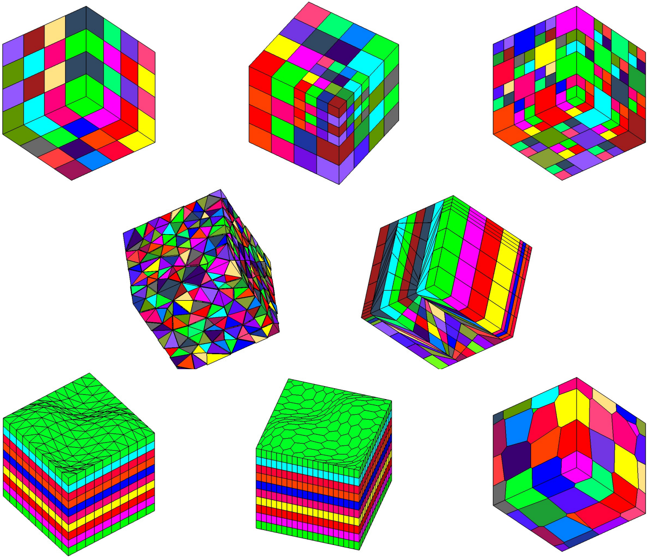
$\tilde{\boldsymbol{p}}^{(3)}$: pressure variables in vertices of dual mesh
and on faces of boundary primal mesh



$$\begin{bmatrix} \mathbb{H}^{(2)}\partial_t + \mathbb{C}[\boldsymbol{u}^{(2)}] & \mathbb{H}^{(2)}\mathbb{D}^{(2,1)} & \tilde{\mathbb{D}}^{(2,3)} \\ \tilde{\mathbb{D}}^{(1,2)}\mathbb{H}^{(2)} & -\nu^{-1}\mathbb{H}^{(1)} & 0 \\ (\tilde{\mathbb{D}}^{(2,3)})^T & 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{u}^{(2)} \\ \nu\boldsymbol{\omega}^{(1)} \\ \tilde{\boldsymbol{p}}^{(3)} \end{bmatrix} = \begin{bmatrix} \text{RHS} \end{bmatrix}$$

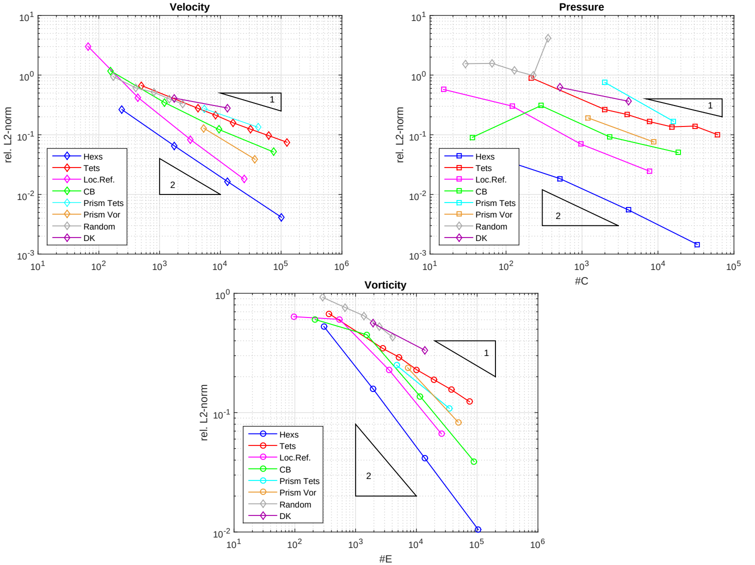
First Numerical verfication on 3D meshes

Difficult meshes (benchmark FVCA-6 2011 and FVCA-8 2017)

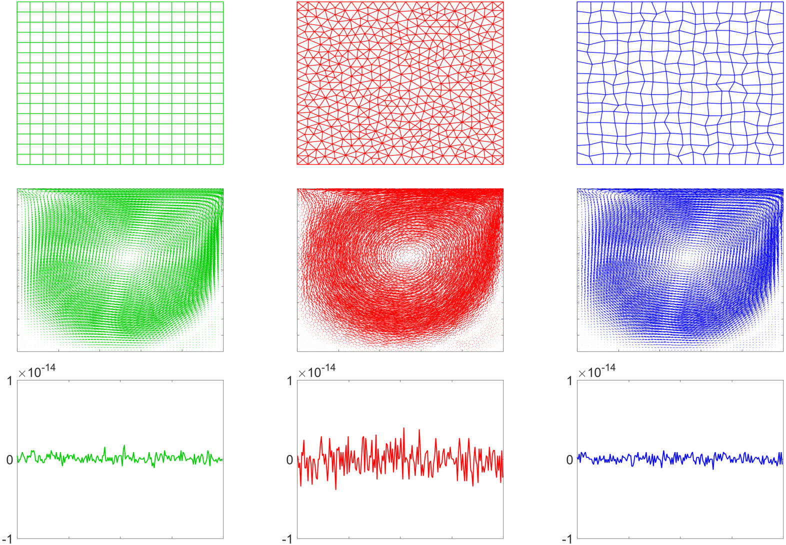


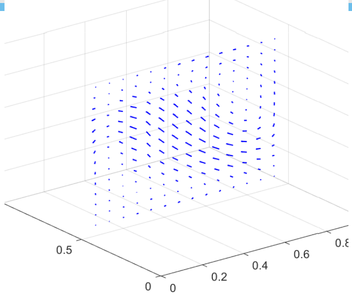
Conservative scheme

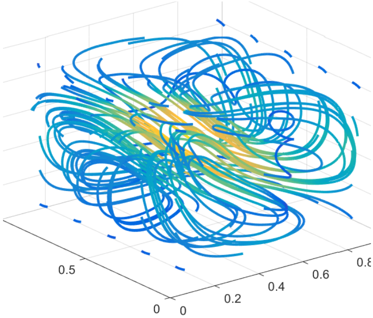
L2-convergence for different benchmark meshes for 3D Taylor-Green (Stokes)

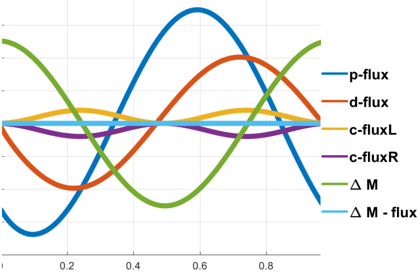


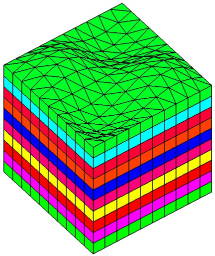
Exact Conservation of Vorticity: *Lid-Driven Cavity - Plot of* $\Omega_{total} - 1$



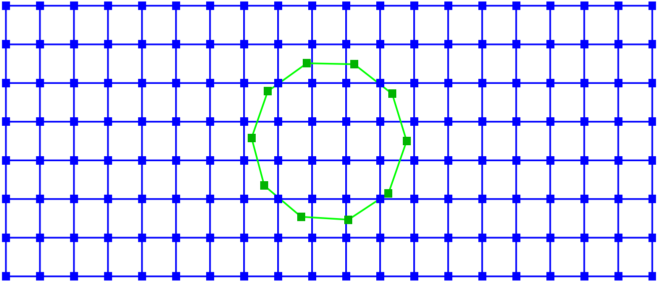

0.5
0
0
0.2
0.4
0.6


0.5
0
0
0.2
0.4
0.6


0.2
0.4
0.6
0.8
p-flux
d-flux
c-fluxL
c-fluxR
Δ M
Δ M - flux



Cut-cell mesh: 2D flow around a monopile



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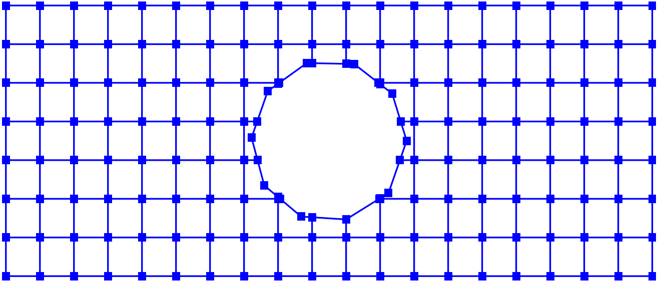

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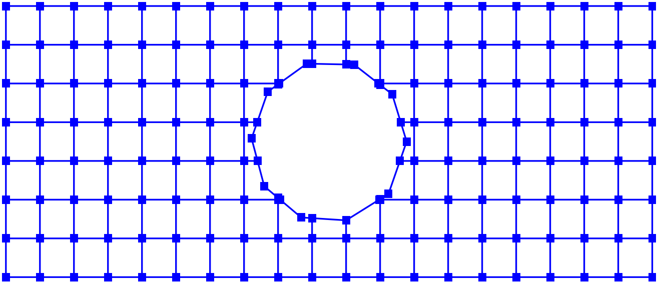

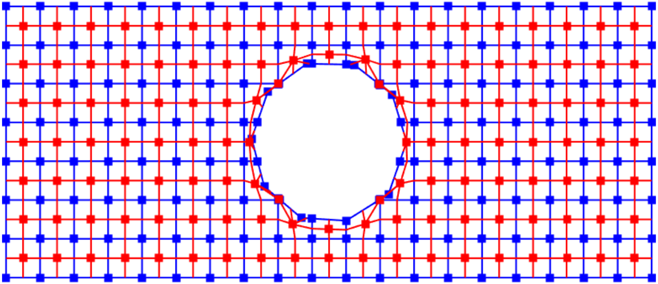

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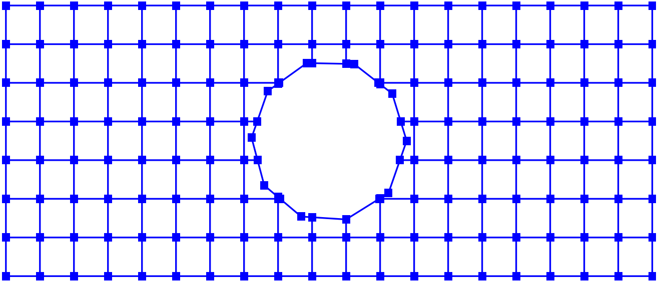

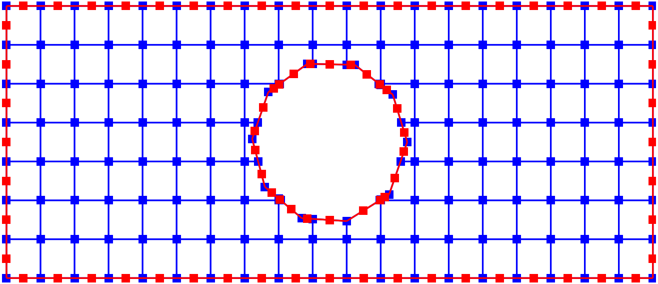

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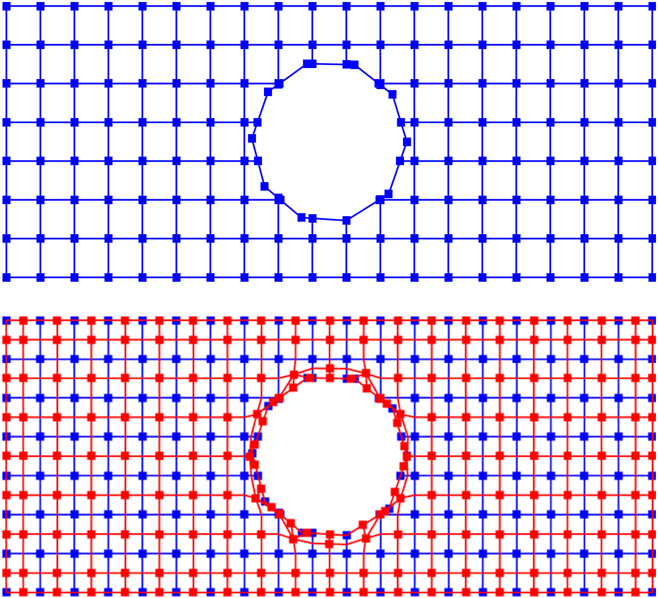

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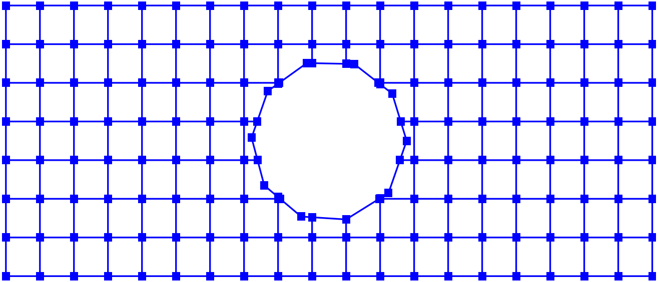
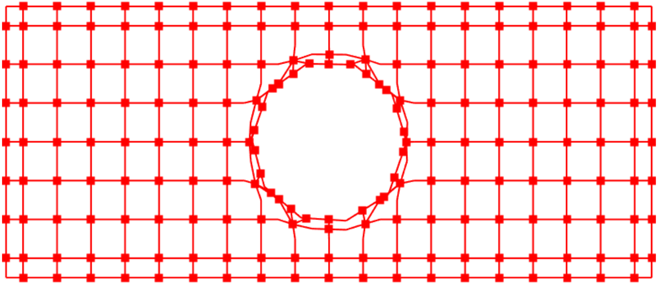
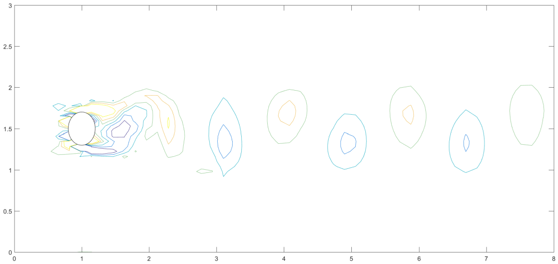
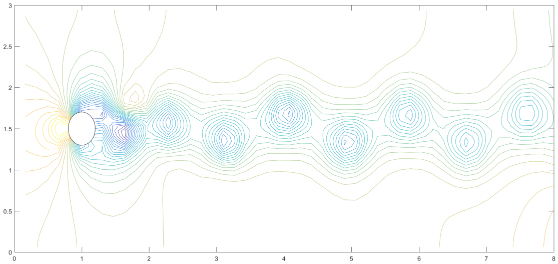






vorticity



pressure