

Bayesian Calibration of Turbulence Models

Using an interpolating surrogate

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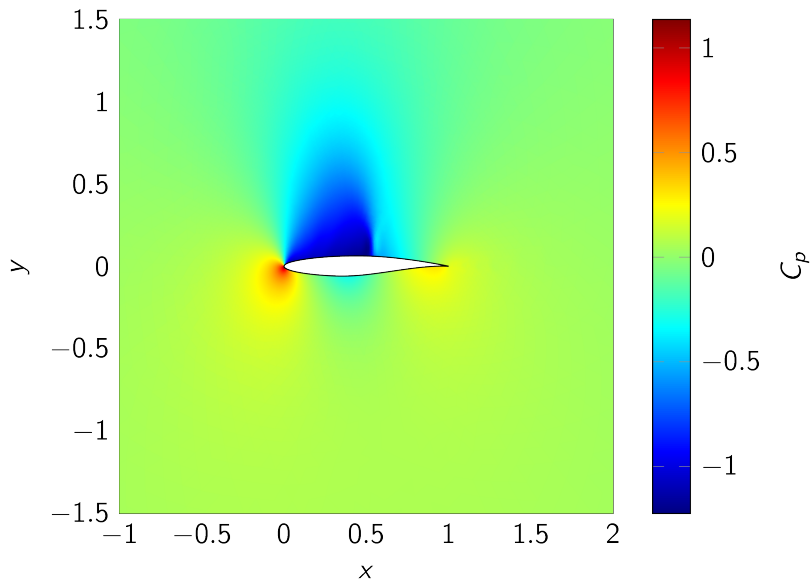


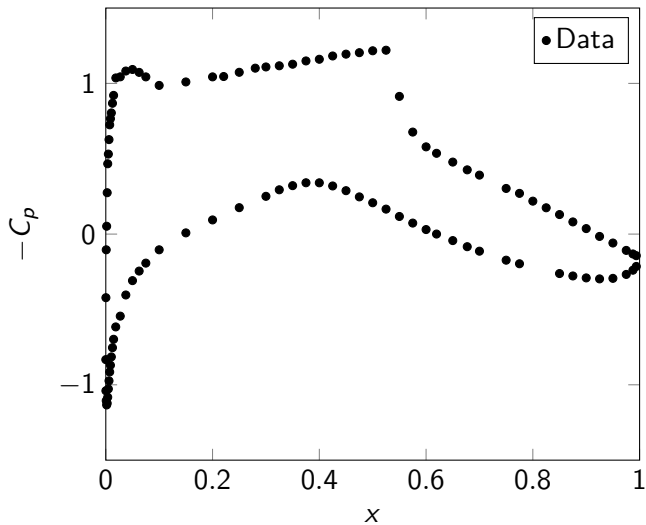
What is the **pressure coefficient** on the airfoil for:

1. $\alpha = 2.92^\circ$
2. $M = 0.725$
3. $\text{Re} = 6.5 \cdot 10^6$

1. Start with **Reynolds-averaged Navier-Stokes**
2. Good turbulence model for airfoils: **Spallart–Almaras**
3. Coefficients:

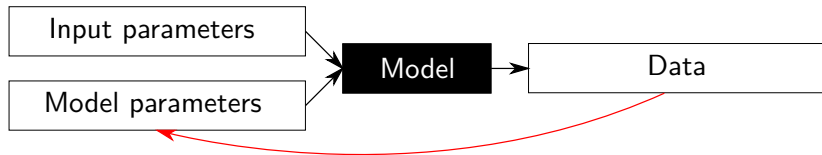
σ	2/3	C_{b1}	0.1355	C_{b2}	0.622
κ	0.41	C_{w2}	0.3	C_{w3}	2.0
C_{t3}	1.2	C_{t4}	0.5	C_{v1}	7.1

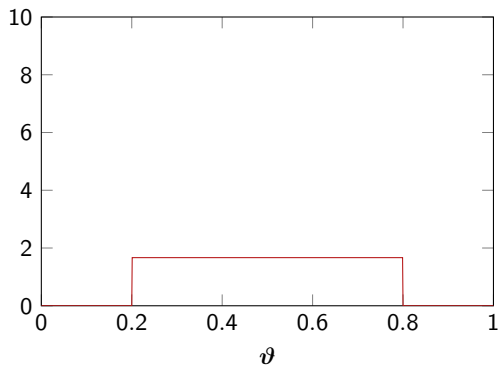




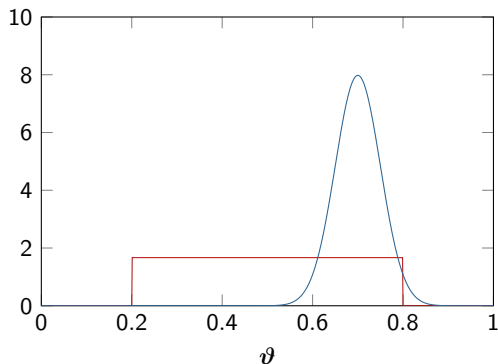






**Prior**

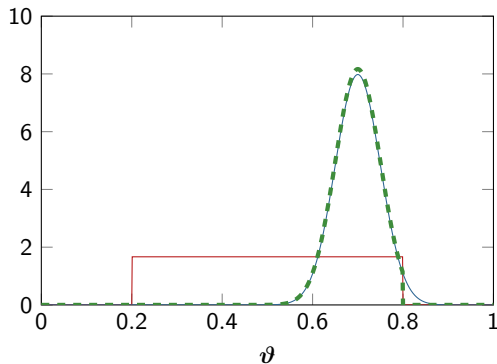
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knowledge, known data

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Likelihood

$p(\mathbf{z} | \vartheta)$:
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Posterior

Bayes' law:
 $p(\vartheta | \mathbf{z}) \propto p(\mathbf{z} | \vartheta)p(\vartheta)$

Strategy:

1. Construct **interpolant** u_N
2. Construct **approximate posterior** $p_N(\vartheta | \mathbf{z})$ with u_N
3. Determine **new node** x_{N+1} with $p_N(\vartheta | \mathbf{z})$
4. **Reconstruct** interpolant u_{N+1} and iterate

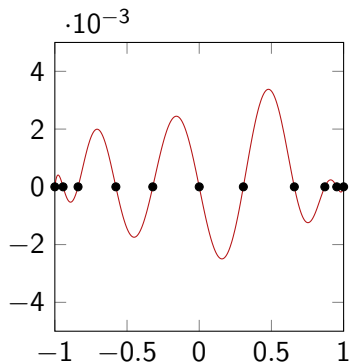
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Question: How to determine next node?

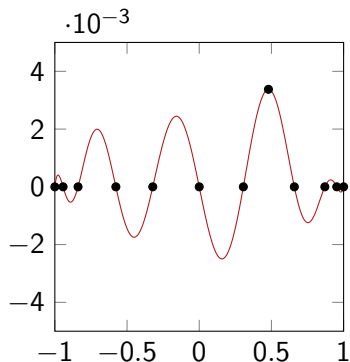
Weighted Leja nodes: let x_0, \dots, x_k be given, then

$$x_{k+1} = \arg \max_x \rho(x) |\ell_k(x)| = \arg \max_x \rho(x) |x-x_0| |x-x_1| \cdots |x-x_k|$$



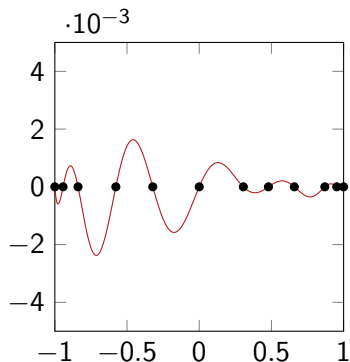
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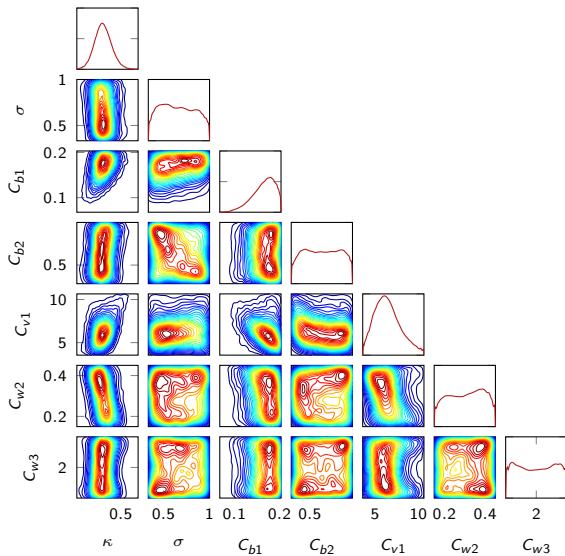


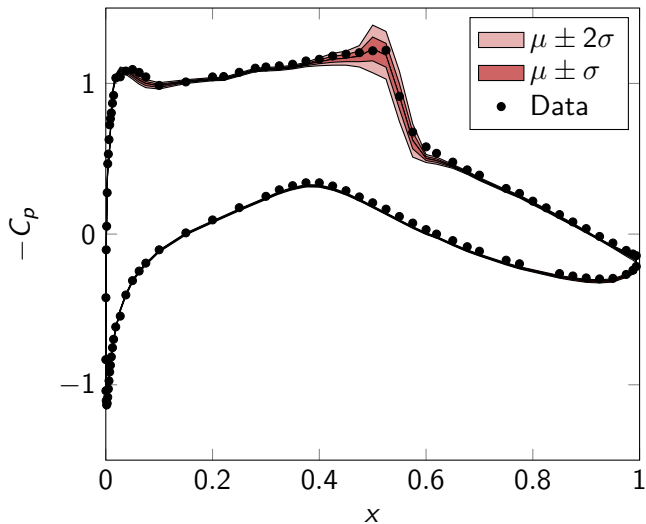


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Future work:

1. Apply Leja nodes to BLADED
2. Obtaining measurement data
3. BLADED high-fidelity vs. low-fidelity?

